First (Recall):

1] For what values of x is $x-3$ **positive**?

greater than zero

$$x-3>0$$
$$+3 \quad +3$$
$$x>3$$

2] For what values of x is $3-x$ **non-negative**?

greater than or equal to zero

R ⩾ G

$$3-x \geq 0$$
$$-3 \quad -3$$
$$(-1)\cdot(-x) \geq (-3)\cdot(-1)$$
$$x \leq 3$$

"warm-up"

3] For what values of x is $ax+b$ **positive**?

$$ax+b>0$$
$$-b \quad -b$$
$$ax>-b$$

two cases arise:

case 1:

$a>0$

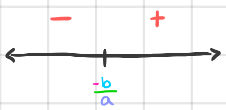
(i.e. a is positive)

$$\frac{ax}{a} > \frac{-b}{a}$$

$$x > \frac{-b}{a}$$

(i.e. to the right of $\frac{-b}{a}$)

ax+b :



case 2:

$a<0$

(i.e. a is negative)

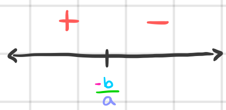
$$\frac{ax}{a} > \frac{-b}{a}$$

since a is negative sign must flip

$$x < \frac{-b}{a}$$

(i.e. to the left of $\frac{-b}{a}$)

ax+b :



Speed round:

For what values of x are the following **positive**?

(i) $3x-2$

(ii) $5x+3$

(iii) $-2x-3$

(iv) $7-x$

(v) $3-2x$

Chapter 3
Section 3.2-3.4
Problem 1. Solve the following inequalites.

(a) $x^2 + 2x + 1 > 0$

$(x+1)(x+1) > 0$

$(x+1)^2 > 0$

$(x+1)^2 = 0$

$x+1 = 0$

$x \neq -1$

$(-\infty, -1) \cup (-1, \infty)$

(b) $10x^2 < 3 - 13x$

$+13x - 3 \quad -3 \quad +13x$

$r \cdot s = (10)(-3)$

$r + s = 13$

$r = 15$

$s = -2$

$10x^2 + 13x - 3 < 0$

$10x^2 + 15x - 2x - 3 < 0$

$5x(2x+3) - (2x+3) < 0$

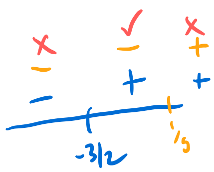
$(2x+3)(5x-1) < 0$

$2x+3 > 0$

$x > \frac{-3}{2}$

$5x - 1 > 0$

$x > \frac{1}{5}$



$(-3/2, 1/5)$

$-3/2 < x < 1/5$

Problem 2. For the following functions graph each function and then determine what values of x the outputs are positive.

(a) $f(x) = -x^2 + 4x + 21$

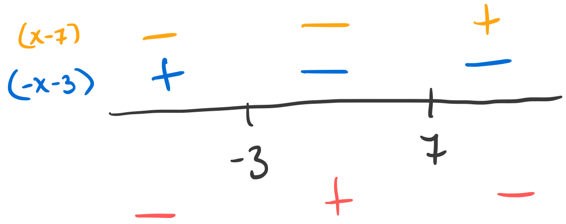
$(-x-3)(x-7) > 0$

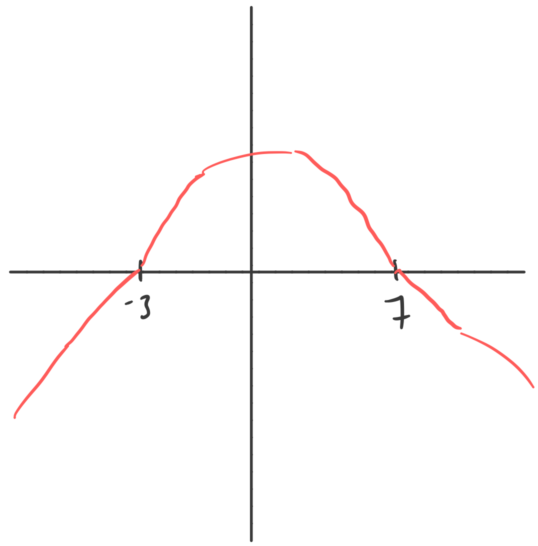
$-x-3 > 0$

$-3 > x$

$x-7 > 0$

$x > 7$





positive: $(-3, 7)$

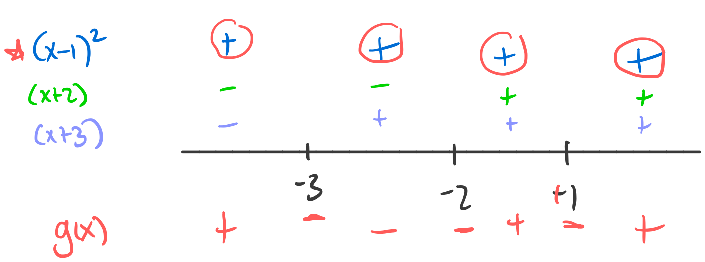
alternatively

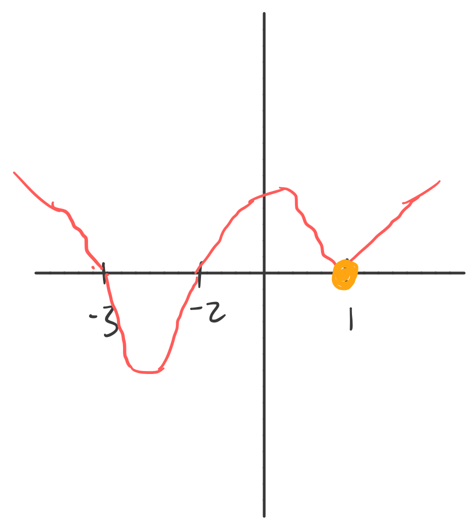
$-3 < x < 7$

(b) $g(x) = (x-1)^2(x+2)(x+3)$

$(x-1)^2 > 0$ $\quad$ $(x+2) > 0$ $\quad$ $(x+3) > 0$

$x \neq 1$ $\quad$ $x > -2$ $\quad$ $x > -3$



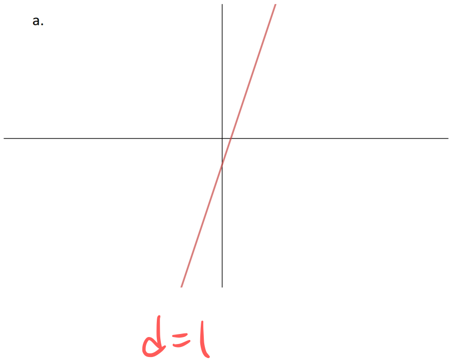


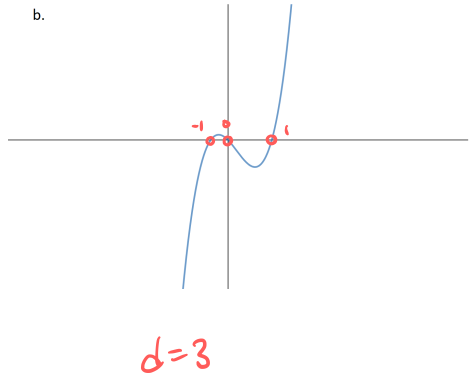
positive: $(-\infty, -3) \cup (-2, -1) \cup (1, \infty)$

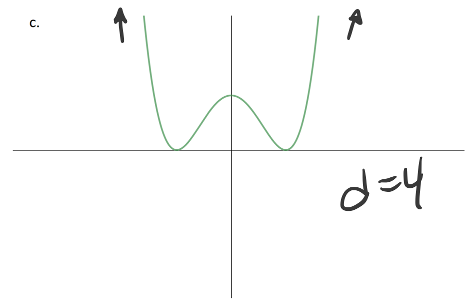
"or"

$x < -3$ or $-2 < x < 1$ or $x > 1$

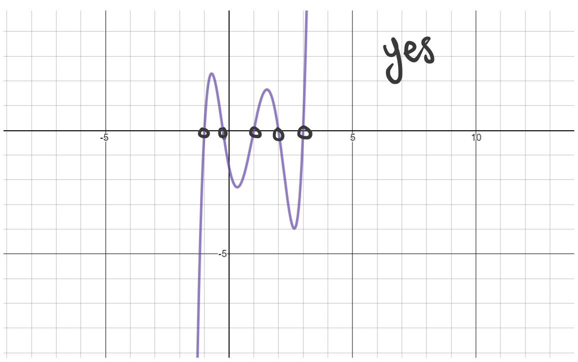
Problem 3. Each graph below represents a polynomial function. For each graph determine a possible degree for the polynomial. What is the lowest possible degree?

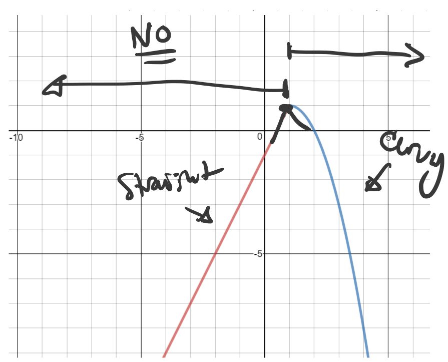


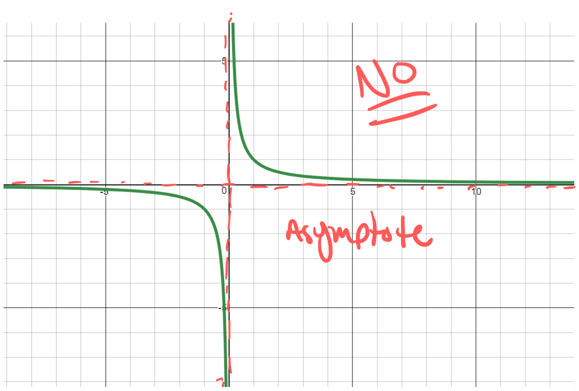


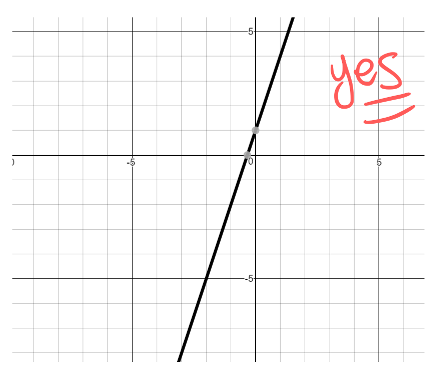


Problem 4. For each of the following graphs determine if the graph can represent a polynomial.







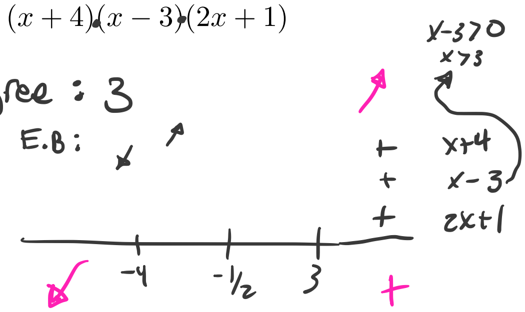


Problem 5. For each of the polynomials below, state the degree and the number of nonzero terms, and describe the end behavior.

(a) $p(x) = (x+4)(x-3)(2x+1)$

degree : 3

E.B : ↙ ↗



(b) $q(x) = 3.7x^3 + 12x + 2x^6$

deg: 6

EB: ↖ ↗

(c) $r(x) = x^5 - 7x^3 + 2x^4 + 1$

degree = 5

EB: ↙ ↗

(d) $s(x) = -3x^5$

$(-1)^5 = -1$

deg: 5

EB: ↖ ↘

Steps:

1. Find the degree

If odd

leading Coefficient:

positive: ↙ ↗

i.e.

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

negative: ↖ ↘

i.e.

$\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -\infty$

If Even

leading Coefficient:

positive: ↖ ↗

i.e.

$\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$

negative: ↙ ↘

i.e.

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = -\infty$

Problem 6. Let $f(x)$ be a polynomial function of degree n, where n is a positive integer.

(a) Suppose n is even. Are the following statements true or false? If true, explain your reasoning. If false, give an example of a polynomial of even degree for which the statement does not hold.

(i) $f(x)$ is an invertible function.

(ii) If $\lim_{x\to-\infty} f(x) = -\infty$, then $\lim_{x\to\infty} f(x) = -\infty$.

(iii) $\lim_{x\to\infty} f(x) = -\infty$.

Problem 7. For the following functions determine all intercepts and the end behavior.

(a) $f(x) = -(x+2)^4(5x+4)$

intercepts: $(-2,0), (\frac{-4}{5},0)$

End behavior: ↖ ↘

it's degree 5 w/ negative leading coefficent

i.e. $\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$

(b) $g(x) = (x-1)^2(x+1)^3(2x-1)$

Intercepts: $(1,0), (-1,0), (\frac{1}{2},0)$

End behavior ↖ ↗

even degree
positive leading coeff.

i.e. $\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \infty$

(c) $h(x) = x^3 - 27$

Intercepts: $(3,0)$

End behavior: ↙ ↗

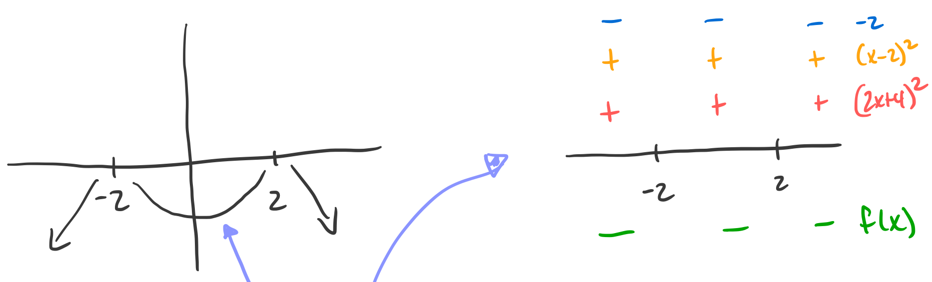
odd degree
positive leading coefficent

i.e. $\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = \infty$

Problem 8. Graph each of the following polynomials. Be sure each graph has accurate intercepts, end behavior, and behavior at roots.

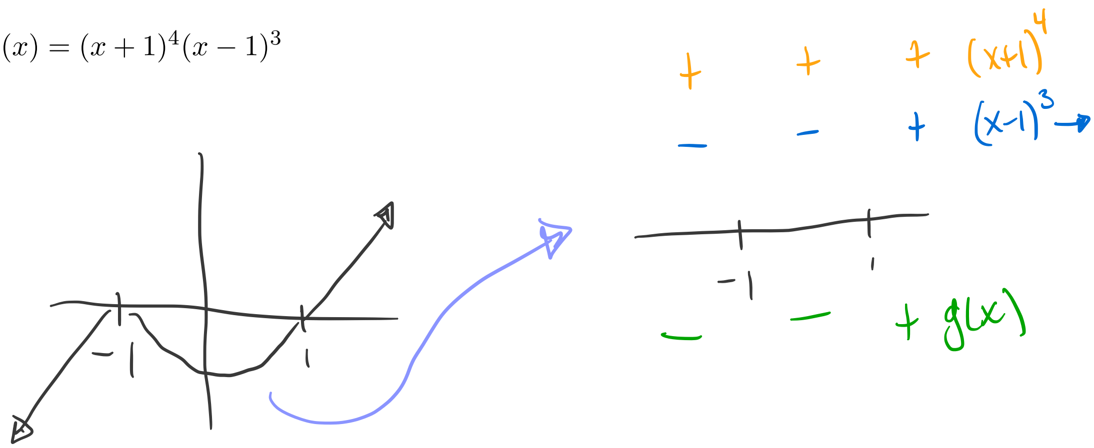
(a) $f(x) = -2(x-2)^2(2x+4)^2$



– I knew the end behavior since even degree and negative leading coefficent

how'd I know this part was negative?

(b) $g(x) = (x+1)^4(x-1)^3$



– End behavior since odd degree and positive leading coefficent

(c) $h(x) = 6x^5 - 5x^4 - 21x^3$

$= x^3(6x^2 - 5x - 21)$ → or factor: $(3x-7)(2x+3)$

x^3 gives intercept $(0,0)$

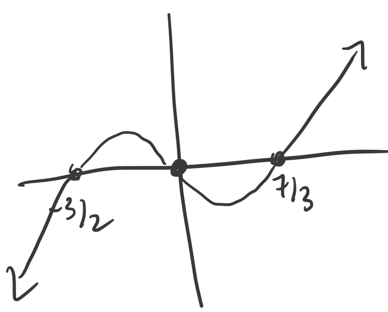
↳ use quadratic eqn

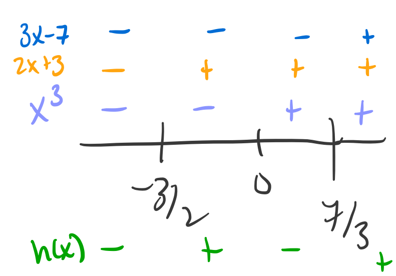
$x = \frac{5 \pm \sqrt{25 - 4(6)(-21)}}{2(6)} = \frac{5 \pm 23}{12}$

so

$\left(\frac{5+23}{12}, 0\right), \left(\frac{5-23}{12}, 0\right)$

$\left(\frac{4}{3}, 0\right), \left(\frac{-3}{2}, 0\right)$





– End behavior odd degree positive leading coeff.